Penrith Selective High School

2018

Higher School Certificate

Trial Examination

Mathematics Extension 1

General instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen.
- NESA approved calculators may be used.
- A reference sheet is provided with this examination paper.
- In questions 11-14, show relevant mathematical reasoning and/or calculations.
- **No** correction tape to be used.

Total marks: 70

Section I – 10 marks (pages 1 - 4)

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 5 - 8)

- Attempt Questions 11-14
- Allow about 1 hour and 45 minutes for this section

	Preliminary	Polynomials	Binomial Theorem	Inverse Trigonometry	Calculus	Probability	Mathematical Induction	
MC Q1-10	**Q1,3 4,5 /4**			**Q6, 7, 9 /3**	**Q2,10 /2**	**Q8 /1**		
Question 11	**a /3**			**f /3**	**b,c,d /8**	**e /1**		
Question 12	**a,b /6**		**d /3**	**c /6**				
Question 13	**d /3**	**b /5**		**a, c /7**				
Question 14					**a,b /8**	**c /4**	**d /3**	**/70**
Total	**/16**	**/5**	**/3**	**/19**	**/18**	**/6**	**/3**	**%**

Student Number:

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

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1 The point P divides the interval from $A(-7,\ 6)$ and $B(4,-6)$ **externally** in the ratio $2:3$.
What is the x-coordinate of P ?

(A) -29

(B) 15

(C) 20

(D) 22

2 What is the value of $\lim_{x \to 0} \frac{sin\ 3x}{5x}$?

(A) 0

(B) $\frac{3}{5}$

(C) 1

(D) $\frac{5}{3}$

3 The Cartesian equation of the tangent, at $t = -2$, to the parabola $x = t - 4$, $y = t^2 + 5$ is:

(A) $4x - y + 15 = 0$

(B) $4x + y - 15 = 0$

(C) $4x - y - 15 = 0$

(D) $4x + y + 15 = 0$

4 What are the asymptotes of $y = \frac{5x}{(x+7)(3x-1)}$?

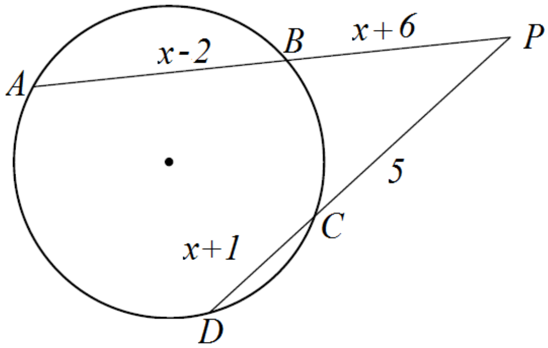
(A) $y = 0,\ x = -7,\ x = \frac{1}{3}$

(B) $y = 0,\ x = 7,\ x = -\frac{1}{3}$

(C) $y = 5,\ x = -7,\ x = -\frac{1}{3}$

(D) $y = 5,\ x = -7,\ x = \frac{1}{3}$

5 Two secants from an external point P cut off intervals on a circle as shown below.



What is the value of x ?

(A) $\frac{1}{2}$

(B) $\frac{1+\sqrt{69}}{2}$

(C) $\frac{3}{2}$

(D) 6

6 Which of the following is the derivative of $tan^{-1}(e^{-2x})$?

(A) $\frac{e^{2x}}{1+e^{2x}}$

(B) $\frac{-e^{-2x}}{1+e^{-2x}}$

(C) $\frac{-2e^{-2x}}{1+e^{4x}}$

(D) $\frac{-2e^{-2x}}{1+e^{-4x}}$

7 Let $|b| \leq 1$. What is the general solution to $\cos \frac{x}{3} = b$?

(A) $k\pi \pm \ cos^{-1} b$

(B) $2k\pi \pm 3\ cos^{-1} b$

(C) $4k\pi \pm \ cos^{-1} b$

(D) $6k\pi \pm 3\ cos^{-1} b$

8 Adrian, Bryson and six friends arrange themselves at random in a circle. What is the probability that Adrian and Bryson are *not* together?

(A) $\frac{1}{5040}$

(B) $\frac{5}{7}$

(C) $\frac{2}{7}$

(D) $\frac{5039}{5040}$

9 The primitive function of $\dfrac{1}{x^2 - 6x + 12}$ is:

(A) $\ln(x^2 - 6x + 12) + C$

(B) $\dfrac{1}{3}\ tan^{-1}\left(\dfrac{x-3}{3}\right) + C$

(C) $\dfrac{1}{\sqrt{3}}\ tan^{-1}\left(\dfrac{x-3}{\sqrt{3}}\right) + C$

(D) $\dfrac{1}{9}\ tan^{-1}\left(\dfrac{x-3}{9}\right) + C$

10 Given that $\dfrac{g'(x)}{g(x)} = 2$, which of the following statements are true?
(Note: C is a constant in each case)

(A) $g(x) = e^x + C$
(B) $g(x) = e^{2x} \times C$
(C) $g(x) = 2\ln x + C$
(D) $g(x) = C \ln x$

End of Section I

Section II

60 marks
Attempt Questions 11-14
Allow about 1 hour and 45 minutes for this section

Answer each question on a **SEPARATE** booklet.

In Questions 11-14, your responses should include relevant mathematical reasonings and/or calculations.

Question 11 (15 marks)

(a) Solve the inequality: $\dfrac{5x}{x-2} \leq 4$ **3**

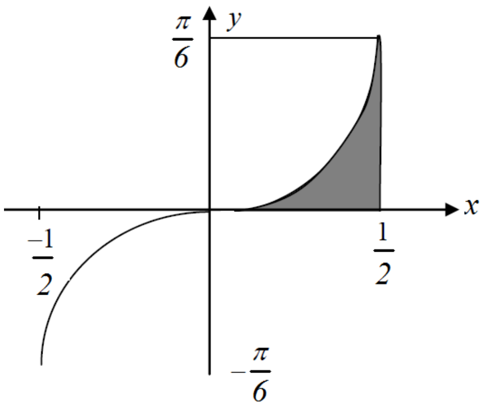
(b) Find $\displaystyle\int cos^2 6x\, dx$ **2**

(c) Evaluate $\displaystyle\int_0^2 \frac{3x}{(3x+1)^2}\,dx$ by using the substitution $u = 3x + 1$. **3**

(d) A circular oil slick lies on the surface of a body of water. Its area is increasing at the rate of $16\ m^2/min$. At what rate is the radius increasing at the time the radius is 5 $metres$ to 2 decimal places ? **3**

(e) Colour blindness affects 8% of all men. What is the expression of the probability that any random sample of 14 males should contain exactly 6 males that are colour blind? **1**

(f) The shaded area shown in the diagram below is bounded by the x-axis, $y = sin^{-1}x$ and the line $x = \frac{1}{2}$.

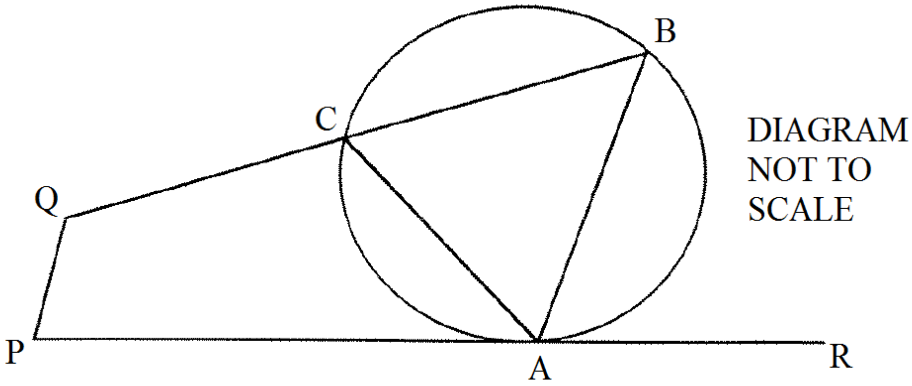


Find the exact value of the shaded area. **3**

End of Question 11

Question 12 (15 marks)

(a) ABC is a triangle inscribed in a circle. PA is a tangent to the circle. PQ is drawn parallel to AB and meets BC produced to Q.
Copy the diagram into your booklet.



Prove $APQC$ is a cyclic quadrilateral. **2**

(b) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $4ay = x^2$ such that the tangents at P and Q intersect at an angle of $45°$. Let T be the point of intersection. The tangent at P is $y = px - ap^2$. (DO NOT PROVE THIS).

i) Show that $p - q = 1 + pq$ **2**

ii) Find the locus of T. **2**

(c) i) Sketch the curve $y = cos^{-1}\left(\frac{x-2}{2}\right)$ **2**

ii) Show that when this curve is rotated about the y-axis , the volume of solid of revolution generated is $6\pi^2$ cubic units. **4**

(d) Show that **3**

$$\binom{n}{0} + 2\binom{n}{1} + 3\binom{n}{2} + \ \ldots\ldots.. + (n+1)\binom{n}{n} = (n+2)2^{n-1}$$

End of Question 12

Question 13 (15 marks)

(a) Find the exact value of $\quad sin\left(cos^{-1}\frac{2}{3}+tan^{-1}\left(-\frac{3}{4}\right)\right)$ **2**

(b) A monic cubic polynomial when divided by x^2+7 leaves a remainder of $x+12$ and when divided by x leaves a remainder of -6.

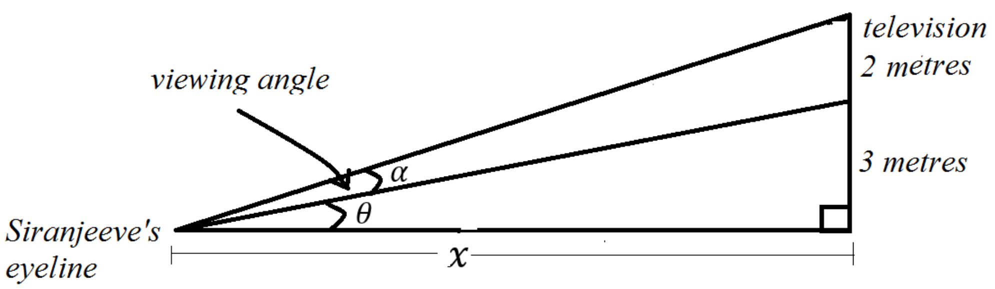
i) Find the polynomial in the form ax^3+bx^2+cx+d. **3**

ii) The polynomial above has a root close to $x=1$. Using one application of Newton's Method, give a better approximation to 2 decimal places. **2**

(c) Siranjeeve buys a 2 metre tall LCD television screen for his cinema room. He mounts it on a vertical wall, placing it so that the base of the television is 3 metres above his horizontal eye line from where he sits in his favourite armchair. Let the distance from his eye to the wall be x metres and the angle from his eye to the top and base of the television be α (the viewing angle).
Let θ be the angle of elevation to the base of the television.

i) Show that the angle of vision α is given by **1**

$$\alpha = tan^{-1}\left(\frac{5}{x}\right) - tan^{-1}\left(\frac{3}{x}\right)$$



ii) Show that for a maximum viewing angle α, $x=\sqrt{15}$ metres. **3**

iii) Hence, find the maximum viewing angle α, to the nearest degree. **1**

(d) Solve for x: $\quad \log_{\frac{1}{2}}\left(\frac{1}{x}\right) \geq \log_2(4x-1)$ **3**

End of Question 13

Question 14 (15 marks)

(a) A certain particle moves along the x-axis according to the equation $t = 4x^2 - 6x + 3$ where x is measured in centimetres and t in seconds. Initially the particle is 1.5 cm to the right of the origin O and moving away from O.

i) Prove that the velocity, $v\ cms^{-1}$ is given by $v = \frac{1}{8x-6}$. **1**

ii) Find an expression for the acceleration, $a\ cms^{-2}$, in terms of x **2**

iii) Find the velocity of the particle when $t = 8$ seconds. **2**

(b) i) Show that the function $T = R + Ae^{-kt}$ is a solution to the differential equation **1**

$$\frac{dT}{dt} = -k(T - R)$$

ii) A metal cake tin is removed from an oven at a temperature of 190°C. If the cake tin takes five minutes to cool to 85°C and the room temperature is 25°C, find the time (to the nearest minute) it takes for the cake tin to cool to 63°C. **2**

(Assume that the cake tin cools at a rate proportional to the difference between the temperature of the cake tin and the temperature of the surrounding air.)

(c) A die is biased so that in any single throw the probability of an odd number is p where p is a constant such that $0 < p < 1, p \neq 0.5$.

i) Show that in six throws of the die the probability of at most one even number is $6p^5 - 5p^6$. **2**

ii) Find the probability that in six throws of the die the product of the scores is even. **2**

(d) Prove by Mathematical Induction

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} \leq 2 - \frac{1}{n}, \text{ for all integers } n \geq 1.$$ **3**

End of Question 14

End of Examination

Find the exact value of the volume of the solid of revolution formed when the region bounded by the curve $y = 3\sec\frac{x}{2}$, the x-axis and the lines $x = 0$ and $x = \pi$ is rotated about the x-axis. **3**

Find the value of k such that $tan^{-1}(k) + tan^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{4}$ **3**

In the expansion of $\left(x^3 + \frac{1}{x}\right)^7$, does the expression contain a constant term? Justify your answer. **2**

A particle is moving in a straight line. At time t seconds it has displacement x $metres$ where from a fixed point O on the line, velocity $v\ ms^{-1}$ given by $v = cos^2 x$ and acceleration $a\ ms^{-2}$. The particle starts at O.

i) Find expressions for a in terms of x for x in terms of t **2**

ii) Sketch the graph of x against t. **1**

ii) Describe the motion of the particle from its initial position to its limiting position. **2**

i) Show that the function $T = R + Ae^{-kt}$ is a solution to the differential equation **1**

$$\frac{dT}{dt} = -k(T - R)$$

ii) A metal cake tin is removed from an oven at a temperature of 190°C. If the cake tin takes ten minutes to cool to 155°C and the room temperature is 25°C, find the time (to the nearest minute) it takes for the cake tin to cool to 90°C. **2**

(Assume that the cake tin cools at a rate proportional to the difference between the temperature of the cake tin and the temperature of the surrounding air.)

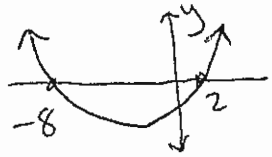
Examination: TRIAL HSC

Level: Extension One

Year: 2018

M.C. 1A 2B 3D 4A 5A 6D 7D 8B 9C 10B

1/2

QUESTION: 11	Markers Comments
(a) $\frac{5x}{x-2} \le 4 \quad x \neq 2$	
$5x(x-2) \le 4(x-2)^2$	
$5x(x-2) - 4(x-2)^2 \le 0$ 1	
$(x-2)[5x - 4(x-2)] \le 0$	* If students
$(x-2)(x+8) \le 0$ 1	wrote $-8 \le x \le 2$ they scored 2 marks.
 $-8 \le x < 2$ 1	
(b) $\int \cos^2 6x\,dx = \frac{1}{2}\int(1+\cos 12x)\,dx$ 1	
$= \frac{1}{2}\left(x + \frac{1}{12}\sin 12x\right) + c$	
$= \frac{1}{2}x + \frac{1}{24}\sin 12x + c$	1
(c) $u = 3x+1$ when $x=2, u=7$	
$\frac{du}{dx} = 3$ when $x=0, u=1$ 1	
$du = 3dx$	
$\int_0^2 \frac{3x}{(3x+1)^2}\,dx = \frac{1}{3}\int_1^7 \frac{u-1}{u^2}\,du$	
$= \frac{1}{3}\int_1^7 \left(\frac{1}{u} - \frac{1}{u^2}\right) du$	
$= \frac{1}{3}\int_1^7 \left(\frac{1}{u} - u^{-2}\right) du$	
$= \frac{1}{3}\left[\ln u + u^{-1}\right]_1^7$ 1	
$= \frac{1}{3}\left[\ln 7 + \frac{1}{7} - (\ln 1 + 1)\right]$	
$= \frac{1}{3}\left(\ln 7 - \frac{6}{7}\right)$ 1	

Examination: Trial

Level: Ext. 1

Year: 2018

2/2

QUESTION:	Markers Comments
(d) $A=\pi r^2$ $\frac{dA}{dr}=2\pi r$ (1) $\frac{dr}{dt}=\frac{dr}{dA}\times\frac{dA}{dt}$ $=\frac{1}{2\pi r}\times 16$ $=\frac{16}{2\pi r}$ when $r=5$, $\frac{dr}{dt}=\frac{16}{10\pi}$ $=\frac{8}{5\pi}$ (1) $=0.51$ ∴ radius is increasing at 0.51 m/min	*A lot of students forgot to find the answer correct (1) to 2dp.
(e) p = men who are colourblind = 0.08 q = men who aren't colourblind = 0.92 P(exactly 6 men are colourblind) $={}^{14}C_6(0.08)^6(0.92)^8$	(1)
(f) Area $=\int_0^{1/2}\sin^{-1}x\,dx$ = Area of rectangle $-\int_0^{\pi/6}\sin x\,dx$	(1)
$=\frac{\pi}{6}\times\frac{1}{2}-\left[-\cos x\right]_0^{\pi/6}$	
$=\frac{\pi}{12}-\left(-\cos\frac{\pi}{6}--\cos 0\right)$	(1)
$=\frac{\pi}{12}-\left(-\frac{\sqrt{3}}{2}+1\right)$ $=\frac{\pi}{12}+\frac{\sqrt{3}}{2}-1$ $=\frac{\pi+6\sqrt{3}-12}{12}$ (1)	
$*\left(\frac{2}{\sqrt{3}}-1\right)$ = 1 mark only $*\ \frac{\pi}{12}+\frac{\sqrt{3}}{2}$ =	2 marks only
$*\left(1-\frac{\sqrt{3}}{2}\right)$ = 2 marks only	

Examination: EXTENSION 1 TRIAL HSC

Level: EXTENSION 1

Year: 2018

QUESTION: 12 | Markers Comments

(a) Let $\angle RAB = \alpha$

$\angle RAB = \angle BCA$ (angle in the alternate segment)

$\angle QCA = 180 - \alpha$ (straight angle)

Let $\angle PAC = \beta$

$\angle PAC = \angle ABC$ (angle in the alternate segment)

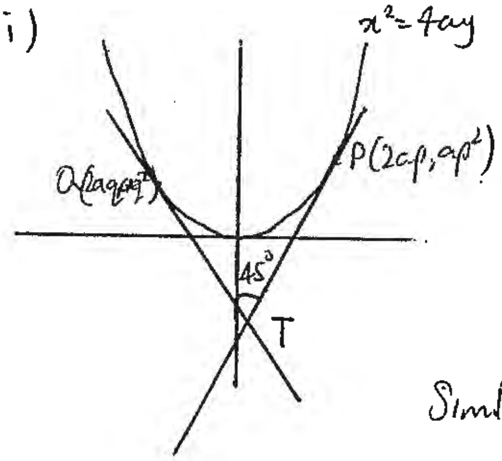
$\angle PQB = 180 - \beta$ (co-interior angles are supplementary $QP \parallel AB$)

$\therefore \angle PAC + \angle QPC$ are supplementary.

$\therefore APQC$ is a cyclic quadrilateral.

Most students did this well

(b) i)



$4ay = x^2$

$y = \frac{x^2}{4a}$

$y' = \frac{x}{2a}$

$m_1 = y'(p)$

$= \frac{2ap}{2a} = p$

Similarly, $m_2 = q$

$\tan 45° = \left|\frac{p-q}{1+pq}\right|$

$1 = \left|\frac{p-q}{1+pq}\right|$

$\therefore p - q = 1 + pq$

Most students were able to use angle between two lines.

ii) The tangent at P is $y = px - ap^2 \ldots (1)$

Similarly at Q: $y = qx - aq^2 \ldots (2)$

(1) − (2): $(p-q)x - a(p^2 - q^2) = 0$

$x = \frac{a\cancel{(p-q)}(p+q)}{\cancel{p-q}} = a(p+q)$

$y = ap(p+q) - ap^2$

$= \cancel{ap^2} + apq - \cancel{ap^2}$

$y = apq$

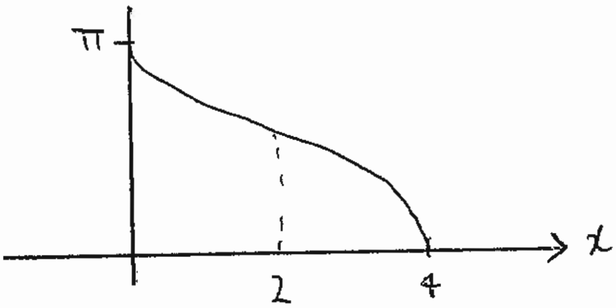
$\therefore p + q = \frac{x}{a}, \; pq = \frac{y}{a}$

Use in (i) for locus

Examination: TRIAL MATHS

Level: EXTENSION 1

Year: 2018.

QUESTION: 12	Markers Comments
b) continued. From (i): $(1+pq)^2 = (p-q)^2$ $= (p+q)^2 - 4pq$ Locus: $\left(1+\frac{y}{a}\right)^2 = \left(\frac{x}{a}\right)^2 - \frac{4y}{a}$ $\therefore a^2 + y^2 + 6ay = x^2$	Only few students were able to find the equation of the locus.
(c) Domain: $-1 \le \frac{x-2}{2} \le 1$, Range $0 \le y \le \pi$. $-2 \le x-2 \le 2$ $0 \le x \le 4$  $\cos y = \frac{x-2}{2}$ $x-2 = 2\cos y$ $x = 2\cos y + 2$ $x^2 = (2\cos y+2)^2$ $= 4\cos^2 y + 8\cos y + 4$	Students were able to get to this step.
$V = \pi\int_a^b x^2\,dy$ $= 4\pi\int_0^{\pi} (\cos^2 y + 2\cos y + 1)\,dy$ $= 4\pi\left[\frac{1}{2}y + \frac{1}{4}\sin 2y + 2\sin y + y\right]_0^{\pi}$ $= 4\pi\left[\left(\frac{\pi}{2} + 0 + 0 + \pi\right) - 0\right]$	Some students could not $\int (\cos^2 y)\,dy$.

$= 4\pi\left(\frac{3\pi}{2}\right)$

$= 6\pi^2$ cubic units.

Examination: EXT 1 MATHS TRIAL HSC

Level: EXTENSION 1

Year: 2018

QUESTION: 12	Markers Comments
Consider $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \dots + \binom{n}{n}x^n$	Students did not do this well.
$\boxed{\text{Multiply by } x:}$	
$x(1+x)^n = \binom{n}{0}x + \binom{n}{1}x^2 + \binom{n}{2}x^3 + \dots + \binom{n}{n}x^{n+1}$	
$\boxed{\text{Differentiating:}}$	
$x(n(1+x)^{n-1}) + (1+x)^n = \binom{n}{0} + 2\binom{n}{1}x + 3\binom{n}{2}x^2 + \dots + (n+1)\binom{n}{n}x^n$	
$(1+x)^{n-1}(xn + (1+x)) = \binom{n}{0} + 2\binom{n}{1}x + 3\binom{n}{2}x^2 + \dots + (n+1)\binom{n}{n}x^n$	
$(1+x)^{n-1}(x(n+1)+1) = \binom{n}{0} + 2\binom{n}{1}x + 3\binom{n}{2}x^2 + \dots + (n+1)\binom{n}{n}x^n$	
$\boxed{\text{Substitute } x=1:}$	
$2^{n-1}(n+2) = \binom{n}{0} + 2\binom{n}{1} + 3\binom{n}{2} + \dots + (n+1)\binom{n}{n}$	
END OF QUESTION 12	

Examination:

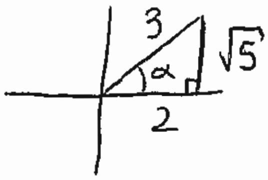
Level:

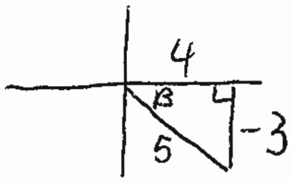
Year:

QUESTION: 15	Markers Comments

a) $\sin\left(\cos^{-1}\frac{2}{3} + \tan^{-1} -\frac{3}{4}\right)$

let $\alpha = \cos^{-1}\frac{2}{3}$ and $\beta = \tan^{-1}\left(-\frac{3}{4}\right)$





so $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{\sqrt{5}}{3} \times \frac{4}{5} + \frac{2}{3} \times \frac{-3}{5}$ — 1 mark for negative sign.

$= \frac{4\sqrt{5}}{15} - \frac{6}{15}$

$= \frac{4\sqrt{5}-6}{15}$

1 mark for final answer.

b) $P(x) = ax^3 + bx^2 + cx + d$

i) but $a = 1$ since monic

$P(0) = 0 + 0 + 0 + d$

$d = -6$

1 mark for $a = 1$, $d = -6$

$$
\begin{array}{r}
x + b \\
x^2+7 \overline{\big) x^3 + bx^2 + cx - 6} \\
-x^3 + 0x^2 + 7x \\
\hline
0 + bx^2 + (c-7)x - 6 \\
-bx^2 + 0x + 7b \\
\hline
0 + (c-7)x - 6 - 7b
\end{array}
$$

1 mark for division of polynomials.

$c - 7 = 1$ $\qquad$ $-6 - 7b = 12$

$c = 8$ $\qquad$ $-7b = 18$

$b = \frac{-18}{7}$

1 mark for $c = 8$, $b = \frac{-18}{7}$

$\therefore P(x) = x^3 - \frac{18}{7}x^2 + 8x - 6$

Part b) was done poorly.

Examination:

Level:

Year:

QUESTION: | **Markers Comments**

b ii) $P(x) = x^3 - \frac{18}{7}x^2 + 8x - 6$

$P'(x) = 3x^2 - \frac{36}{7}x + 8$

$P(1) = \frac{3}{7}$

$P'(1) = \frac{41}{7}$

$x_2 = x_1 - \frac{P(1)}{P'(1)}$

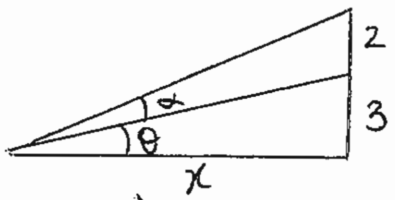
$= 1 - \frac{3/7}{41/7}$

$= 1 - \frac{3}{41}$

$= \frac{38}{41} = 0.92$

Newton's method carry from previous error.

13c)



i) $\tan(\alpha+\theta) = \frac{5}{x}$ $\quad \tan(\theta) = \frac{3}{x}$

$\theta + \alpha - \theta = \tan^{-1}\left(\frac{5}{x}\right) - \tan^{-1}\frac{3}{x}$

1 mark.

ii) $\frac{d\alpha}{dx} = \frac{1}{1+\left(\frac{5}{x}\right)^2} \times -5x^{-2} - \frac{1}{\left(1+\left(\frac{3}{x}\right)^2\right)} \times -3x^{-2}$

$= -\frac{\frac{5}{x^2}}{1+\frac{25}{x^2}} + \frac{\frac{3}{x^2}}{1+9/x^2}$

$= -\frac{5/x^2}{\left(\frac{x^2+25}{x^2}\right)} + \frac{3/x^2}{\left(\frac{x^2+9}{x^2}\right)}$

Examination:

Level:

Year:

QUESTION:	Markers Comments
$\frac{d\alpha}{dx} = -\frac{5}{x^2+25} + \frac{3}{x^2+9}$ $\frac{d\alpha}{dx} = 0$ for max $\frac{5}{x^2+25} = \frac{3}{x^2+9}$ $5x^2+45 = 3x^2+75$ $2x^2 = 30$ $x^2 = 15$ $x = \pm\sqrt{15}$ $\quad x>0$, $\therefore x=\sqrt{15}$ $\begin{array}{c\|c\|c\|c} x & 3.5 & \sqrt{15} & 3.9 \\ \hline \frac{d\alpha}{dx} & 0.01 & 0 & -0.00195 \\ & + & 0 & - \end{array}$ $\quad \therefore x=\sqrt{15}$ is a max.	1 mark setting up $\frac{d\alpha}{dx}$ correctly. 1 mark for $x=\sqrt{15}$ 1 mark for testing $x=\sqrt{15}$ is max.
iii) $\alpha = \tan^{-1}\left(\frac{5}{x}\right) - \tan^{-1}\left(\frac{3}{x}\right)$ $= \tan^{-1}\left(\frac{5}{\sqrt{15}}\right) - \tan^{-1}\left(\frac{3}{\sqrt{15}}\right)$ $= 14^\circ$.	1 mark for answer.
d) $\frac{\log_2 1/x}{\log_2 1/2} \geq \log_2(4x-1)$ $\frac{\log_2 x^{-1}}{\log_2 2^{-1}} \geq \log_2(4x-1)$ $\frac{\cancel{-}\log_2 x}{\cancel{-}1} \geq \log_2(4x-1)$ $x \geq 4x-1$ $3x \leq \frac{1}{3}$ But $x > \frac{1}{4}$ since $4x-1>0$ $\therefore 1/4 < x \leq 1/3$.	1 mark change the base. 1 mark for $x \leq 1/3$ 1 mark for testing $x > \frac{1}{4}$

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QUESTION: 14	Markers Comments
a) $t = 4x^2 - 6x + 3$	
(i) $\frac{dt}{dx} = 8x - 6$	well done
$v = \frac{dx}{dt} = \frac{1}{8x-6}$	
(ii) $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$	Most of the students did it well
$= \frac{d}{dx}\left(\frac{1}{2}(8x-6)^{-2}\right)$	
$= \frac{1}{2} \times -2(8x-6)^{-3} \times 8$	very few wrote $(8x-6)^{-1}$ instead of $(8x-6)^{-3}$
$= \frac{-8}{(8x-6)^3}$	
(iii) $t = 8$ sec.	
$4x^2 - 6x + 3 = 8$	
$4x^2 - 6x - 5 = 0$	
$x = \frac{6 \pm \sqrt{36+80}}{8}$	
$= \frac{6 \pm \sqrt{116}}{8}$	good attempt
$= \frac{3 \pm \sqrt{29}}{4}$	
$x = \frac{3+\sqrt{29}}{4}$ as $a > 0$ of the particle always moves to the right and velocity is never zero	good to see students stating reasons for choosing positive value of x.
$v = \frac{1}{8\left(\frac{3+\sqrt{29}}{4}\right) - 6}$	
$= \frac{1}{\cancel{6} + 2\sqrt{29} - \cancel{6}} = \frac{1}{2\sqrt{29}}$ (OR 0.093 cm/s)	

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Year: 2018

QUESTION:	Markers Comments
b) (i) $T = R + Ae^{-kt}$	good attempt
$\frac{dT}{dt} = -kAe^{-kt}$	
$= -k(T-R)$	
$\therefore$ $T = R + Ae^{-kt}$ is a solution to the differential equation.	
(ii) $t=0$, $T=190^\circ C$, $t=5$, $T=85^\circ$, $R=25^\circ$	Most of the students found A, k correctly
$190 = 25 + Ae^0$ $(e^0=1)$	
$\boxed{A = 165}$, Also, $85 = 25 + 165e^{-5k}$	
$\frac{\cancel{60}\ 4}{\cancel{165}\ 11} = e^{-5k}$	
$-5k = \ln\left(\frac{4}{11}\right)$	
$\boxed{k = -\frac{1}{5}\ln\left(\frac{4}{11}\right)}$	
$63 = 25 + 165e^{-kt}$	
$\cancel{63}\ 25 = 165e^{-kt}$	
$\frac{\cancel{5}\ 38\ \cancel{28}}{\cancel{165}\ \cancel{35}} = e^{-kt}$	
$t = -\frac{1}{k}\ln\left\|\frac{38}{165}\right\|$	well done
$= 7.25..$ minutes	
$= 7$ min (to the nearest minute)	

Examination: HSC – Trial Examination

Level: Extension 1 – 1

Year: 2018

QUESTION: 14	Markers Comments
c) $P(\text{odd}) = P$, $P(\text{even}) = 1-P$	
at most one even = Probability of 0 even + prob of 1 even	
(i) $= {}^6C_6\, P^6 (1-P)^0 + {}^6C_5\, P^5 (1-P)$	good attempt
$= P^6 + 6P^5 - 6P^6$	
$= 6P^5 - 5P^6$	
(ii) Product of scores is even, when it is atleast one even.	Most of the students could not do this part
$1 - {}^6C_6\, P^6 (1-P)^0$	
$= 1 - P^6$.	
d) $1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} \leq 2 - \frac{1}{n}$	
<u>step 1</u>: $n=1$, LHS $= 1$, RHS $= 2 - \frac{1}{1} = 1$	well done for steps 1 and 2.
LHS = RHS, the result is true for $n=1$.	
<u>step 2</u>: Let the result be true for $n=k$	
$1 + \frac{1}{2^2} + \cdots + \frac{1}{k^2} \leq 2 - \frac{1}{k}$	
<u>step 3</u> To show that the result is true for $n=k+1$	
i.e. $1 + \frac{1}{2^2} + \cdots + \frac{1}{k^2} + \frac{1}{(k+1)^2} \leq 2 - \frac{1}{k+1}$	

Examination:

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QUESTION: 14

Markers Comments

LHS.

$$1+\frac{1}{2^2}+ \text{——} + \frac{1}{k^2}+\frac{1}{(k+1)^2} \leq 2-\frac{1}{k}+\frac{1}{(k+1)^2}$$

(using the assumption)

Most of the students were lost in proving the inequality step 3

In order to show that

$$1+\frac{1}{2^2}+ \text{——} + \frac{1}{(k+1)^2} \leq 2-\frac{1}{k+1}$$

enough to show that $2-\frac{1}{k}+\frac{1}{(k+1)^2} \leq 2-\frac{1}{k+1}$

i.e. $\frac{1}{k+1}-\frac{1}{k}+\frac{1}{(k+1)^2} \leq 0$

$$\frac{k(k+1)-(k+1)^2+k}{(k+1)^2} \leq 0$$

i.e. $\frac{\cancel{k^2}+\cancel{k}-\cancel{k^2}-2\cancel{k}-1+\cancel{k}}{(k+1)^2} \leq 0$

i.e. $\frac{-1}{(k+1)^2} \leq 0$ which is true.

∴ using the Principal of Mathematical Induction, the property is true for all $n \geq 1$